Direct SAT-Based Cryptanalysis of Some Symmetric Ciphers

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Outline

1. Cryptanalysis of symmetric ciphers
2. Feistel transformation and DES algorithm
3. Boolean encoding
4. Experiments and future work
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Symmetric ciphers

Algorithms
- Feistel Network 1972,
- DES - Data Encryption Standard, 1974, 56-bits key,
- Feistel based algorithms - MISTY1, Skipjack, Twofish, Blowfish, Camellia, CAST-128, FEAL, ICE, LOKI97, Lucifer, MARS, MAGENTA, RC5, TEA, Twofish, XTEA, GOST 28147-89.

Main operations
- xor,
- permutations,
- bits rotations,
- S-boxes.
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Methods of cryptanalysis

Knowledge point of view
- ciphertext-only attack,
- known-plaintext/ciphertext attack,
- chosen-plaintext attack,
- chosen-ciphertext attack.

Computational point of view
- statistical cryptanalysis,
- brute force,
- differential cryptanalysis,
- linear cryptanalysis,
- SAT-based approach.
SAT based methods

Verification methods

- software testing,
- simulations,
- formal model verification.

SAT based methods

- translating system and investigated property into a boolean propositional formula,
- testing satisfiability using SAT solvers.
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**Verification methods**
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**SAT based methods**
- translating system and investigated property into a boolean propositional formula,
- testing satisfiability using SAT solvers.
Formulas used in this-type verification are translated into Conjunction Normal Form (CNF - conjunction of clauses), for example

\[(p_1 \lor \neg p_2 \lor \neg p_5) \land (\neg p_2 \lor p_3 \lor \neg p_4) \land (p_3 \lor \neg p_4 \lor p_5)\]

Exponential translation algorithm that preserves equivalence, but linear algorithm that preserve satisfiability.

SAT solvers

- SAT Race conferences,
- MiniSAT, BerkMin, CryptoMiniSAT and many others.
SAT

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SAT based methods in security

SAT in security


Feistel network

- Introduced by Horst Feistel for IBM Lab in 1972.
- Block cipher.
- Main idea of Feistel network is used in DES, IDEA and many others ciphers.
Feistel network

plaintext

\[ L, R \]

\[ \text{xor} \]

\[ K \]

\[ \text{xor} \]

\[ L', R' \]

\[ \text{Ciphertext} \]
### Feistel equations

#### Ciphertext

1. \( L' = R, \)
2. \( R' = L \text{ xor} (R \text{ xor} K). \)

#### xor properties

1. \( x \text{ xor} x = 0, \)
2. \( x \text{ xor} 0 = x, \)
3. \( x \text{ xor} (y \text{ xor} z) = (x \text{ xor} y) \text{ xor} z. \)

#### Decrypting

1. \( L' = R, \)
2. \( R' \text{ xor} (L' \text{ xor} K) = L \text{ xor} (R \text{ xor} K) \text{ xor} (R \text{ xor} K) = L. \)
Other DES components

Components

1. permutations,
2. bits rotations,
3. S-boxes

S-box

S-box is a nonlinear component, it is a matrix with 16 columns and 4 rows, where each of rows contains different permutation of \{0, 1, 2, \ldots 15\}.
Other DES components

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S-box

S-box is a nonlinear component, it is a matrix with 16 columns and 4 rows, where each of rows contains different permutation of \{0, 1, 2, \ldots 15\}. 
One round

Let \((p_1, \ldots, p_{64})\) be a plaintext vector, where \(p_i \in \{0, 1\}\), for \(i = 1, \ldots, 64\) and \((k_1, \ldots, k_{32})\) be a key vector, where \(k_j \in \{0, 1\}\), for \(j = 1, \ldots, 32\). We denote by \((c_1, \ldots, c_{64})\) a cipher vector, where \(c_s \in \{0, 1\}\), for \(s = 1, \ldots, 64\).

Encoding formula for one round

\[
\Phi^1_F : \bigwedge_{i=1}^{32} (c_i \leftrightarrow p_{i+32}) \land \bigwedge_{i=1}^{32} [c_{i+32} \leftrightarrow (p_i \text{ xor } p_{i+32} \text{ xor } k_i)]
\]
Feistel encoding

Many rounds

Let \((p_1^1, \ldots, p_{64}^1)\) be a plaintext vector, where \(p_i^1 \in \{0, 1\}\), for \(i = 1, \ldots, 64\) and \((k_1, \ldots, k_{32})\) be a key vector, where \(k_j \in \{0, 1\}\), for \(j = 1, \ldots, 32\). We denote by \((c_1^j, \ldots, c_{64}^j)\) a cipher vector after \(j\) rounds, where \(c_s^j \in \{0, 1\}\), for \(s = 1, \ldots, 64\).

Encoding formula for \(j\) rounds

\[
\Phi_F^j : \bigwedge_{i=1}^{32} \bigwedge_{s=1}^{j} (c^s_i \leftrightarrow p^s_{i+32}) \land \bigwedge_{i=1}^{32} \bigwedge_{s=1}^{j} [c^s_{i+32} \leftrightarrow (p^s_i \oplus p^s_{i+32} \oplus k_i)] \land \bigwedge_{i=1}^{64} \bigwedge_{s=1}^{j-1} (p^s_i \leftrightarrow c^s_i). \]
Consider $P$ - the initial permutation function of DES. Let $(p_1, \ldots, p_{64})$ be a sequence of variables representing the plaintext bits.

Denote by $(q_1, \ldots, q_{64})$ a sequence of variables representing the block bits after permutation $P$.

We can encode $P$ as the following formula:

$$\bigwedge_{i=1}^{64} (p_i \Leftrightarrow q_{P(i)}) .$$

In a such way, we can encode all the permutations, expansions, reductions, and rotations of DES.
We can consider each S-box as a function of type
\[ S_{box} : \{0, 1\}^6 \rightarrow \{0, 1\}^4. \]
We denote a vector \((x_1, \ldots, x_6)\) by \(\overline{x}\) and by \(S_{box}^k(\overline{x})\) the \(k\)-th coordinate of value \(S_{box}(\overline{x})\), for \(k = 1, 2, 3, 4\).

We can encode each S-box as the following Boolean formula:

\[
\Phi_{S_{box}} : \bigwedge_{\overline{x} \in \{0,1\}^6} \left( \bigwedge_{i=1}^{6} (\neg)^{1-x_i} p_i \Rightarrow \bigwedge_{j=1}^{4} (\neg)^{1-S_{box}^j(\overline{x})} q_j \right),
\]

where \((p_1, \ldots, p_6)\) is the input vector of S-box and \((q_1, \ldots, q_4)\) the output one. Additionally, by \((\neg)^0 p\) and \((\neg)^1 p\) we mean \(p\) and \(\neg p\), respectively.
Algorithm

Our cryptanalysis procedure

1. encoding a single round of the cipher considered as a Boolean propositional formula;
2. automatically generating of the formula encoding a iterated desired number of rounds (the whole cipher);
3. converting the formula obtained its CNF;
4. (randomly) choosing a plaintext and the key vector as a 0, 1 valuation of the variables representing them in the formula;
5. inserting the chosen valuation into the formula;
6. calculating the corresponding ciphertext using an appropriate key and insert it into the formula;
7. using SAT-solver to find a satisfying valuation, including a valuation of the key variables.
Parallelisation of the process of encoding formula generation. An acceleration about 15%
Optimisation

Optimising the formula obtained by removing all not necessary equivalences. We can decrease the number of variables used sometimes by 50% and the number of clauses by 10%.

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Variables</th>
<th>Variables optimised</th>
<th>Clauses</th>
<th>Clauses optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1024</td>
<td>568</td>
<td>10496</td>
<td>9472</td>
</tr>
<tr>
<td>8</td>
<td>1976</td>
<td>1016</td>
<td>20866</td>
<td>18944</td>
</tr>
<tr>
<td>16</td>
<td>3768</td>
<td>1912</td>
<td>41601</td>
<td>37888</td>
</tr>
</tbody>
</table>
Parallelisation

- Parallel realization of translating formula into a CNF form. 10% speedup in the translation time.

\[
(A \implies B) \iff (C \implies D)
\]

Thread 1  Thread 2  Thread 3  Thread 4
Parallel application of SAT-solver using $2^n$ cores.

key = <k1, k2, k3, k4, \ldots k56, k57, k58>

Thread 1 : <0, 0, k3, k4, \ldots k56, k57, k58>

Thread 2 : <1, 0, k3, k4, \ldots k56, k57, k58>

Thread 3 : <0, 1, k3, k4, \ldots k56, k57, k58>

Thread 4 : <1, 1, k3, k4, \ldots k56, k57, k58>
Optimisation and parallelisation

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Time (s.)</th>
<th>Time with parallelisation (s.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.052</td>
<td>0.036</td>
</tr>
<tr>
<td>4</td>
<td>34.686</td>
<td>6.542</td>
</tr>
<tr>
<td>5</td>
<td>12762</td>
<td>2438</td>
</tr>
<tr>
<td>6 + 20 key bits</td>
<td>47.539</td>
<td>2.216</td>
</tr>
</tbody>
</table>

Parallelisation

In the last row of Table, for six rounds of DES with added valuations of 20 key bits, the whole key was solved in 2.2 secs with all our optimisation and parallelisation. In the same case but without the improvements presented above, the whole key was solved in 145 secs. The best result known so far in SAT-based cryptanalysis for this case was 68 secs.
Future work

- Other methods of formulas optimization.
- Parametrized cryptanalysis with many pairs plaintext/ciphertext.
- Testing several SAT-solvers.
- Testing other ciphers like KAZUMI, RC family or recently introduced hash functions like Grostl, Grain128.
- Create dedicated SAT-solver for cryptanalysis.
That is all :)
Thank You