INTRODUCTION

We investigate queueing systems with demands of random space requirements and limited buffer space, in which queueing or sojourn time are limited by some constant value. For such systems, in the case of exponentially distributed service time and Poisson entry, we obtain the steady-state demands number distribution and probability of demands losing. In our work, we study queueing systems in which demands are also “impatient”. In other words, they can leave the system during their waiting in the queue, or even during their servicing. Such systems are the models of some real processes. E.g., systems of information transmission often deal with the process of messages information reduction.

2. Models description

Consider the M/M/1/n system queueing system with identical servers and FIFO service discipline. Let be the intensity of demands entrance flow, be the parameter of service time. Each demand has some random volume which does not depend on the volumes of other demands nor on the demand arriving epoch.

Let be the demand volume distribution function and be the sum of volumes of all demands present in the system at time instant . The values of the process are limited by the constant value (buffer space capacity). Let us denote by the number of demands present in the system at time . Let a demand having the volume arrive to the system at epoch . Then, it will be accepted to the system if and otherwise the demand will be lost. In opposite case, the demand will be lost and as . If is the epoch when a demand of volume leaves the system, we have

We will assume that the system load is finite ( ). Our goal is to determine the distribution of the stationary number of entries that are present in the system and the probability of reporting loss due to these restrictions.

3. Queueing system with limited buffer space and limited queueing time

3.1. Process and characteristics

Let denote the length of time interval from the moment to the moment when the -th demand leaves the queue (starts its service or is lost) . Let be the volume of the -th demand. It is clear that

The system behavior is described by the following Markov process:

We characterize this process by the following functions:

For we introduce the following functions:

Assume that at least one of the values , , and is finite. Therefore, for , the steady state exists for the system under consideration, i.e., , and and in the sense of a weak convergence, where and are the steady-state number of demands present in the system, their steady-state total volume and steady-state residual time to leave the system for -th demand, respectively.

We obtain the following steady-state analogies of the functions:

For we obtain:


3.2. Steady-state demands number distribution and loss probability

In our work, we write differential equations for the above steady-state functions. From these equations we obtain the steady-state characteristics:

Therefore, for such we have:

Now, for the functions we have:

whereas we obtain for the steady-state demands number distribution:

Let be the event that an arbitrary arriving demand is accepted to the system and served completely. The probability of this event can be calculated as follows:

Let be the event that the request that came to the system stationary will not be lost at the time of its arrival and will continue to be completely served. - average number of occupied service devices (positions). It is clear that

Then the probability that the requests will be lost at the time of arrival or not fully served is

3. Conclusions

We obtain formulae for demands number distribution and loss probability for the queueing systems with constant limitations of the demand total volume and queueing time. The obtained formulae are not generally convenient for precise calculation, but the calculation is possible in some special cases (e.g., when a demand volume has gamma, uniform or exponential distribution, then the probability of its loss is given by the figures 1a-1c).

Fig. 1. Loss probability for .

\[ P(\text{loss}) = 1 - P(A) = 1 - \left(1 - \frac{\mu K}{\rho + \rho K(\mu + \sigma)}\right)^n. \]